Background, Research Base, and Field Testing for *Math Expressions*

Background

Math Expressions is a program based on the research results of the *Children's Math Worlds Research Project* (*CMW Project*). Both the program and the research combine a focus on conceptual understanding with opportunities to develop fluency with problem solving and computation. Dr. Karen C. Fuson, now Professor Emeritus of Learning Sciences at Northwestern University, Evanston, Illinois, is the Principal Investigator of the *CMW Project* and author of ***Math Expressions*.**

In this complete Kindergarten through Grade 5 curriculum, grades K and 1 consist of completely integrated units, and for grades 2–5, longer units that focus on word problems, computation, algebra, and data are followed by shorter units that focus on geometry or measurement and apply previously learned mathematics. This organization is a result of teacher and student feedback during the *CMW Project*. At each grade level, the program includes a two-volume Teacher's Guide, two consumable Student Activity Books, two consumable Homework and Remembering Workbooks, a Teacher's Resource Book, an Assessment Guide, MathBoards, and various manipulatives and other learning materials.

Math Expressions Philosophy

Math Expressions was developed to meet the national need for a balanced program that could

- combine the focus on understanding of reform math programs and the focus on skills of more traditional approaches
- use an approach that emphasized in-depth sustained learning of ambitious core grade-level concepts rather than a spiral curriculum that does not allow for mastery and fluency
- expand the types of word problems to those solved by other countries
- use math drawings made by students and research-based visual representations in each math domain to support student understanding and class discussion of mathematical thinking
- find and use computational algorithms that relate easily to the present common U.S. algorithms but can be understood and used more easily by students
- start where students are, continually elicit their thinking, but provide visual and linguistic supports for understanding that all children use

- bring all children to fluency in the core computational topics that form the central part of the elementary curriculum while still doing important parts of other mathematical topics

The Research Base for ***Math Expressions***

The *Children's Math Worlds Project* received funding from the National Science Foundation: two major research grants and a materials development grant to conduct research on how students could learn more deeply in various domains and to develop curriculum materials in addition and subtraction computation and word problems. Additional financial support was provided by grants from the James S. McDonnell Foundation and the Dwight D. Eisenhower Professional Development Program.

The project began in 1993 and funding from the grants ended in 2003. The major research was carried out in Spanish-speaking as well as English-speaking classrooms. The materials development work was carried out in urban high-poverty classrooms as well as in middle-income and wealthy classrooms. Most of the research was done in the Chicago area, along with a substantial number of schools in the Phoenix area, and some in California, Connecticut, and Virginia. All research focused on the methods of teaching in various domains, and the formative results have been folded into revisions in every year, with most grade levels revised at least five times in response to teacher feedback and observations of students in classrooms.

The *Children's Math Worlds Research Project* carried out ten years of research, working with teachers, students, parents, and administrators to identify learning approaches accessible to all children. The project identified accessible computational methods that all children can understand and carry out independently; designed visual learning supports on which students can draw to show math concepts (the MathBoards); and developed learning paths that move from these visual supports to sketched math drawings linked to math notation and then to independent but meaningful computation. Word problems were continually intertwined with all computational learning, and students from all backgrounds solved an ambitious and carefully chosen trajectory of word problem types across the grades. Through the research, effective language and solution methods that students used in our classrooms (e.g., division is "unmultiplying") were gathered and woven into the learning materials. Continual focus on sense-making and helping and explaining within a classroom community facilitated language development, competence, and confidence. A few core classroom structures (for example, Quick Practice with Student Leaders, Math Talk with Solve and Explain) enabled students to focus on the mathematics rather than on management of materials and student activities.

Among the key research factors influencing the program are the power of student drawings to express math thinking and support math discussions, the kinds of algorithms that fit student thinking and relate easily to current methods, developmental sequences of student strategies in math domains, student conceptual language, visual representations to support understanding, and types of word problems. Features of the program that reflect this research include coherent in-depth curricular learning paths, ongoing interactions between individual and whole-class learning, differentiated instruction within whole class activities, and research-based models, strategies, and algorithms that help all students.

Data Summary of Studies Focusing On Particular Core Concept Domains

In all studies focusing on target research areas from kindergarten through grade 5, *CMW* students exceeded the performance of comparison groups by 10% to 40% and often were comparable to or exceeded the performance of students 1 to 4 years ahead. The target research topics were learning in kindergarten, powerful and general finger and mental methods for adding and subtracting numbers to 20, place value concepts and multidigit addition and subtraction methods, word problems, multiplication and division, and fractions, ratio, proportion. Specific research articles are listed in the list of publications by topic area.

On simple word problems *CMW* first graders did as well as or better than Japanese and Taiwanese children, and they did considerably better than children in the United States from a range of backgrounds who received traditional instruction (mean difference 30%) (tasks from Stigler, Lee, & Stevenson, 1990). *CMW* children did particularly better on subtraction problems, showing only a 10% lower performance on subtraction vs. addition compared to other samples that showed subtraction performance from 22% to 38% lower than addition. On money tasks first and second graders did considerably better than children in a small university city who received traditional instruction (mean differences of 35%), and the first graders did as well as the traditional second graders (Wood & Cobb, 1989). On various tasks assessing understanding of tens groupings and place value, the *CMW* first graders did better than first graders using the reform curriculum Everyday Mathematics (EM) (20% better) and considerably better than children using traditional curricula (40% better) and than children in Taiwan and Japan (30%).

On a simple addition word problem the performance of *CMW* first graders was 20% higher than that of children using a traditional text. On a simple subtraction word problem the performance of *CMW* first graders was 20% higher than that of children using EM, 25% higher than children from Taiwan and Japan, and 45% higher than U.S. children using a traditional text. On word problems of moderate difficulty (unknown change, unknown part, compare and equalize, multiplication), *CMW* first graders did considerably better than the traditional first graders (mean difference of 25%); they also did considerably better than traditional second graders on a 2-step problem (23% difference) and as well as the second graders on an unknown change problem. The performance of *CMW* first graders on tasks interpreting tables and graphs was close to that of Japanese children (5% difference), was somewhat better than the EM children and children from Taiwan (mean differences of 13% and 15%), and was considerably better than that of the EM children in Chicago and U.S. children receiving traditional instruction (a mean difference of 27% and 25%). On a task involving 2-digit addition with regrouping, the scores of *CMW* first graders were 20% higher than those for EM children, 28% higher than those for Taiwanese and Japanese children, and 45% higher than those for children using traditional textbooks. On numerical addition and subtraction problems, *CMW* first-graders were correct on at least 90% of problems with totals? 10, were 10% higher than standardized test norms for addition problems in the teens, and were 17% higher than standardized test norms for subtraction problems in the teens (e.g., 15 – 7) for second graders.

The *CMW* second graders were 20% higher than standardized test norms for subtraction problems in the teens (e.g., 15 – 8). They did considerably better on 2-digit addition than did U.S. children using a traditional text (80% versus 50% correct), better than grade 2 standardized test norms (80% versus 60%), and a bit better than EM children (80% versus 70%). On 2-digit subtraction with regrouping, *CMW* students scored 21% higher than did U.S. children using a traditional text, 33% higher than grade 2 standardized test norms, and 25% higher than EM children. On 3-digit subtraction with regrouping, *CMW* students scored 14% higher than did U.S. children using a traditional text, 25% higher than grade 2 standardized test norms, and 21% higher than EM children. On non-simple word problems *CMW* students scored 13% higher than a heterogeneous sample of students using traditional texts.

The *CMW* third graders were 24% higher than standardized test norms for 3-digit addition problems, 30% higher on 2-digit subtraction problems, and 24% higher on 3-digit subtraction problems. On NAEP items, *CMW* students scored 30% higher than the national NAEP sample on a place-value item, 11% higher on 2-digit addition and subtraction, 18% higher on a subtraction number story, 70% higher on the area of a rectangle, 60% higher on the perimeter of a rectangle, and 14% higher on bar graph items. On division and 2-step word problems, *CMW* students scored 25% higher than a heterogeneous sample using traditional textbooks, and 30% higher on division computation than that sample. Urban *CMW* third graders did well on single-digit multiplication and division computation, scoring 17% higher than the national standardized test norms. On Grade 4 level extra information or 2-step word problems
from Stigler, Lee, & Stevenson, 1990, the *CMW* third graders were equivalent to U.S. fifth graders using traditional textbooks.

On-Line Resources:
Content Standards: http://www.cde.ca.gov/be/st/ss/documents/mathstandard.pdf

Houghton Mifflin Math Expressions: http://www.eduplace.com/math/mthexp/

Student Support Materials: http://www.iusd.org/student_resources/Mathematics.html

Word Problems: http://thinkingblocks.com/

Interactive Dictionary: http://www.amathsdictionaryforkids.com/